# – – Differentiation – –

## Definition of Derivative

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

find the equation of the tangent line at $x = 3$ of $f(x) = 5x^2 + 4$

$\lim_{h \to 0} \frac{5(x+h)^2 + 4 - (5x^2+4)}{h}$

$\lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 + 4 - 5x^2 - 4}{h}$ *

$\lim_{h \to 0} 10x + 5h$

$f'(x) = 10x \to f'(3) = 30$

$y - 49 = 30(x - 3)$

*if you did the equation right, all the remaining terms should be divisible by h

*$f'(x)$ is the derivative, $f'(30)$ is the slope of the tangent line $x = 3$

## Power Rule

find the equation of the tangent line at $x = 3$ of $f(x) = 5x^2 + 4$

$f'(x) = (2)5x^{2-1}$

$f'(x) = 10x$

$f'(3) = 30$

$y - 49 = 30(x - 3)$

*always use this rule unless your forced to use the definition of a derivative

## Chain Rule

find the equation of the tangent line at $x = 1$ of $f(x) = (x^3 + 4)^2$

**let $u = x^3 + 4$, so $f'(x) = 2u^1(u')$**

$f'(x) = 2(x^3 + 4)^{2-1}(3x^2 + 0)$

$f'(x) = 6x^2(x^3 + 4)$

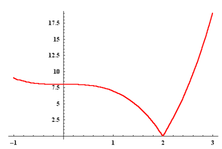
$f'(1) = 6(1)^2[(1)^3 + 4)] = 30$

$y - 25 = 30(x - 1)$

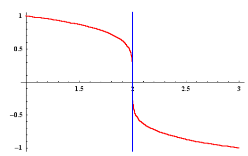
technically: $\frac{dy}{dx} f(g(x)) = f'(g(x))(g'(x)$

* always ask yourself if you need to use chain rule, in calculus it is almost always yes

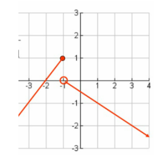
## Derivative Does Not Exist When →



sharp turns



vertical tangent line



not continuous at that point

## Product Rule

$\frac{dy}{dx} f(x) \cdot g(x) = f(x)[g'(x)] + g(x)[f'(x)]$

$\frac{dy}{dx} 5x^2(x+1) = 5x^2(1) + (x+1)(10x) = 5x^2 + 10x^2 + 10x = 15x^2 + 10x$

## Quotient Rule

$\frac{dy}{dx} \frac{f(x)}{g(x)} = \frac{g(x)[f'(x)] - f(x)[g'(x)]}{[g(x)]^2}$

$\frac{dy}{dx} \frac{5x+5}{x+2} = \frac{(x+2)(5) - (5x+5)(1)}{(x+2)^2} = \frac{5x + 10 - 5x - 5}{x^2 + 4x + 4} = \frac{5}{x^2 + 4x + 4}$

## ← Trig Function Derivatives

sec → sec → tan

csc → –csc → cot

$\frac{dy}{dx} \sin u = u' \cos u$

$\frac{dy}{dx} \cos u = -u' \sin u$

## Interpreting Problems →

average velocity: find the slope over given time interval

instantaneous rate of change: find the velocity at the time being asked using the derivative

## Implicit Differentiation

find the derivative of $xy^2 + 5y = 10$

$x(2y) \frac{dy}{dx} + y^2(1) + 5 \frac{dy}{dx} = 0$

$\frac{dy}{dx}(2xy + 5) = -y^2$

$\frac{dy}{dx} = -y^2/(2xy + 5)$

* whenever you find the derivative of y, always put in $\frac{dy}{dx}$ right after it

## Related Rates

1. write out the equation for the situation you are dealing with (ex. $a^2 + b^2 = c^2$ for a right triangle, $\pi r^2 h$ for a cylinder)
2. if any measurements remain constant throughout the process, plug it in
3. take the derivative of your equation. put in a $\frac{d(variable)}{dt}$ for every variable you take the derivative of. that represents the rate of which that variable is changing at
4. plug in your values and solve
5. if you don't have some values, you may need to set up a ratio between two variables and substitute a known variable in for the unknown one (ex. the radius is always twice the height, so you can plug in r = 2h in the equation if you don't know r)

* it helps to draw a picture of what the scenario will always look like and what the scenario looks like at the time that is being asked