-- Integration --

Riemann Sums

Left

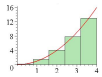
* the height of the rectangle is equal to the y-value on the left
* this will be an overestimate if the function is decreasing
* this will be an underestimate if the function is increasing

Right

* the height of the rectangle is equal to the y-value on the right
* this will be an overestimate if the function is increasing
* this will be an underestimate if the function is decreasing

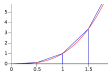
Midpoint

* the height of the rectangle is equal to the y-value of the x-value that is in the middle of the already given x-values



Trapezoidal

* the y-values of each of the x-values are the two bases of the trapezoid. the height is the distance bw the two x-values



Indefinite Integrals

Solving Integrals

for polynomials: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

for trig functions:

$\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

sec → sec → tan

csc → -csc → cot

if the integrand is two trig functions multiplied together and they're in the same row in the "trick", the other trig function in the row is the answer.

*remember that if the answer starts with a c, there's a - in front of it

$\int 5x\sqrt[3]{1-x^2}\,dx$

$u = 1 - x^2$

so the $du = -2x\,dx$

$\int 5x\sqrt[3]{1-x^2}\,dx \rightarrow 5\int x\sqrt[3]{u}\,dx \rightarrow 5\left(\frac{-1}{2}\right)\int -2x\sqrt[3]{u}\,dx \rightarrow \frac{-5}{2}\int \sqrt[3]{u}\,du \rightarrow \frac{-5}{2}\int u^{\frac{1}{3}}\,du$

$\rightarrow \frac{-5}{2}\left(\frac{u^{4/3}}{4/3}\right) + C \rightarrow \frac{-15}{8}(1-x^2)^{4/3} + C$

U-Substitution

1. determine if the integrand can be simplified so that you can use the "power rule"
2. if not, try to determine what your u is. remember that the derivative of u must be to the same degree as the other "stuff" in the integrand
3. substitute in the u if needed. manipulate the rest of the integrand so you can substitute du. if you want, manipulate the integrand into a form that you are comfortable integrating.
4. integrate as normal, then "unsubstitute" the u

$\int \frac{\sin x}{\cos^3 x}\,dx$

$\int \frac{\sin x}{\cos x}\left(\frac{1}{\cos^2 x}\right)dx \rightarrow \int \tan x\sec^2 x\,dx$

$u = \sec^2 x$

so the $du = 2\sec x(\sec x\tan x)dx = 2\sec^2 x\tan x\,dx$

$\int \tan x\sec^2 x\,dx \rightarrow \frac{1}{2}\int 2\tan x\sec^2 x\,dx \rightarrow \frac{1}{2}\int du$

$\rightarrow \frac{1}{2}u + C \rightarrow \frac{1}{2}\sec^2 x + C$

Definite Integrals

Normal

1. Integrate as normal using the "power rule". If you can't, see the next column for u-sub directions
2. Plug in the upper limit into the integrated function. Then plug in the lower limit. Subtract the upper limit answer - lower limit answer

$\int_1^3 (3x^2 - 4x + 2)dx$

$x^3 - 2x^2 + 2x\,]_1^3 \rightarrow$

$[(3)^3 - 2(3)^2 + 2(3)] - [(1)^3 - 2(1)^2 + 2(1)] =$

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U-Substitution

(see table above this to review u-substitution)

1. Determine the u and du. Plug the upper limit into the x-position in the u formula, and your answer is the new upper limit. Do the same with lower limit.
2. Integrate as normal, see the first column for a detailed explanation.

$\int_0^{\pi/4} \frac{\sin x}{\cos^3 x}\,dx \rightarrow \int_0^{\pi/4} \tan x\sec^2 x\,dx$

$u = \sec^2 x$, so $du = 2\sec^2 x\tan x\,dx$

upper limit: $u = \sec^2\left(\frac{\pi}{4}\right) = 2$

lower limit: $u = \sec^2(0) = 1$

$\int_0^{\pi/4} \frac{\sin x}{\cos^3 x}\,dx = \int_1^2 du$

$\rightarrow \frac{1}{2}u\,]_1^2 \rightarrow \frac{1}{2}[2-1] \rightarrow \frac{1}{2}$

Theorems/Properties

Integral Properties

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

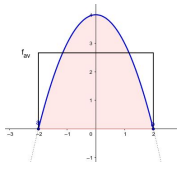
even functions: $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

odd functions: $\int_{-a}^{a} f(x)dx = 0$

Average Value

if f is integrable on the interval [a,b], the

average value = $\frac{1}{b-a}\int_a^b f(x)dx$



the average value is the y-value/height of a rectangle with the same area as the answer to the original integral

2nd Fundamental Theorem

$\frac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)dt\right] = f(h(x))h'(x) - f(g(x))g'(x)$

Ex.

$\frac{d}{dx}\left[\int_{3x}^{5x^2} (6t)dt\right] = 6(5x^2)(10x) - 6(3x)(3) =$

$300x^3 - 54x$