-- Applying Integration --

Area of a Region Between Two Curves

$f(x) = -x^2 + 4x + 1$ $\quad g(x) = x + 1$

x-boundaries

$-x^2 + 4x + 1 = x + 1 \rightarrow 0 = x^2 - 3x$

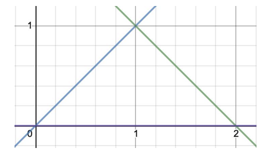
$0 = x(x - 3) \rightarrow x = 0 \text{ and } x = 3$

$$\int_0^3 [(-x^2 + 4x + 1) - (x + 1)]dx$$

$$\int_0^3 (-x^2 + 3x)dx = 4.5$$

1. find the points of intersections by graphing or setting the equations equal to each other
2. set up your integral based on the intersecting points. If one function is on top and one is on the bottom, use x – boundaries, and do top – bottom. If one function is on the right and one function is on the left, use y–boundaries, and do right – left
3. Simplify the definite integral to find your answer

$y = x$ $\quad y = 2 - x$ $\quad y = 0$



y-boundaries

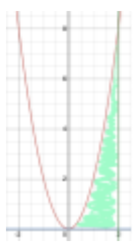
$y = x \rightarrow x = y$ $\qquad y = 2 - x \rightarrow x = 2 - y$

$$\int_0^1 [(2 - y) - (y)]dy \rightarrow \int_0^1 (2 - 2y)dy = 1$$

Disk Method

$y = 2x^2, \quad y = 0, \quad x = 2$

around the x-axis



intersects at (0,0)
intersects at (2,8)

$$\pi \int_0^2 (2x^2)^2 dx = \frac{128}{5}\pi$$

$$\pi \int_a^b [R(x)]^2 dx \text{ or } \pi \int_a^b [R(y)]^2 dy$$

1. if they're not given to you, find the intersection points either algebraically or by graphing
2. confirm that the axis of rotation is touching the shaded area. If not, use the washer method
3. set up your integral. if the axis of rotation is horizontal, use dx's, x–boundaries, and do top – bottom. If the axis of rotation is vertical, use dy's, y–boundaries, and do right – left.
4. solve the integral

around the line x = 2

$0 = 2x^2 \rightarrow x = 0 \rightarrow$ intersects at (0,0)

$y = 2(2^2) \rightarrow y = 8 \rightarrow$ intersects at (2,8)

$$y = 2x^2 \rightarrow x = \sqrt{\frac{y}{2}}$$

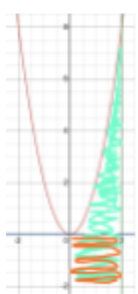
$$\pi \int_0^8 (2 - \sqrt{\tfrac{y}{2}})^2 dy = \pi \int_0^8 (4 - 4\sqrt{\tfrac{y}{2}} + \tfrac{y}{2})dy$$

$$= \frac{16}{3}\pi$$

Washer Method

$y = 2x^2, \quad y = 0, \quad x = 2$

around the line y = -2



intersects at (0,0)
intersects at (2,8)

$R(x) = 2x^2 + 2$ $\quad r(x) = 2$

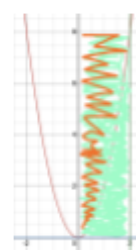
$$\pi \int_0^2 [(2x^2 + 2)^2 - (2)^2]dx$$

$$\frac{704}{15}\pi$$

$$\pi \int_a^b [(R(x))^2 - (r(x))^2]dx \text{ or } \pi \int_a^b [(R(y))^2 - (r(y))^2]dy$$

1. if they're not given to you, find the intersection points either algebraically or by graphing
2. confirm that the axis of rotation is not touching the shaded area. If it is, use the disk method
3. determine your big radius and your little radius. try to imagine being able to plug in a value into your radius equations, and the answer is the distance from the border function to the axis of rotation.
4. set up your integral. if the axis of rotation is horizontal, use dx's and x–boundaries.. If the axis of rotation is vertical, use dy's and y–boundaries
5. solve the integral

around the y-axis



intersects at (0,0)
intersects at (2,8)

$R(y) = 2$ $\quad r(y) = \sqrt{\frac{y}{2}}$

$$y = 2x^2 \rightarrow x = \sqrt{\frac{y}{2}}$$

$$\pi \int_0^8 [(2)^2 - (\sqrt{\tfrac{y}{2}})^2]dy$$

$$16\pi$$

Cross Sections

$y = x^3, \quad y = 0, \quad x = 1$

squares perpendicular to x-axis

intersects at (0,0) and (1,1)

area of a square $= s^2 = (x^3 - 0)^2$

$$\int_0^1 (x^3 - 0)^2 dx = \frac{1}{4}$$

1. find the boundaries of the area
2. set up an equation to find the area of the shape. then replace the variables with the function.
3. know that perpendicular to x-axis is dx's and x-boundaries, while perpendicular to the y-axis is dy's and y-boundaries
4. put your area function into an integral with the necessary boundaries. solve

semicircles perpendicular to y-axis

intersects at (0,0) and (1,1)

$y = x^3 \rightarrow x = \sqrt[3]{y}$

A of semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(1 - \sqrt[3]{y})^2$

$$\frac{1}{2}\pi \int_0^1 (1 - \sqrt[3]{y})^2 dy = \frac{1}{20}\pi$$