-- Polynomials and Rational Functions --

Simplifying

$$\frac{\frac{3}{x-1}-\frac{4}{3}}{\frac{x}{3x^2-3}+\frac{2}{x+1}} = \frac{\frac{9-4(x-1)}{3(x-1)}}{\frac{x(x+1)+2(3x^2-3)}{(3x^2-3)(x+1)}} =$$

$$\frac{\frac{9-4x+4}{3(x-1)}}{\frac{x^2+x+6x^2-6}{(3x^2-3)(x+1)}} = \frac{\frac{13-4x}{3(x-1)}}{\frac{7x^2+x-6}{(3x^2-3)(x+1)}} = \frac{13-4x}{3(x-1)} \bullet \frac{(3)(x^2-1)(x+1)}{7x^2+x-6}$$

$$= 13-4x \bullet \frac{(x+1)^2}{7x^2+x-6} = \frac{(13-4x)(x+1)^2}{7x^2+x-6}\ ;\ D: \mathbb{R}$$

1. Combine the fractions in the numerator & denominator by using a common denominator (multiply by fancy form of 1)
2. Multiply out the numerators in both the numerator and denominator (note this does not apply every time, it depends on the function & whether there is anything that can be cancelled out w/o foiling **you can figure out whether you need to do it by checking if any part of the functions are the same and can be cancelled out before multiplying out
3. Take the denominator & flip it over and multiply it w/ the numerator; now we can cancel stuff
4. Once everything has been cancelled & can not be simplified further, then combine the functions
5. Domain is determined by holes, zeros in denominator function, & and anywhere where graph is present

Average Rate of Change (ARC)

$\frac{f(b)-f(a)}{b-a}$; f(x)= $3x^2+1$; find arc from [0,4]

$$\frac{f(4)-f(0)}{4-0} = \frac{49-1}{4} = 12$$

✻ Fancy form of 1 is like $\frac{x+1}{x+1}$, still technically 1 because it's the same function/value on the top and bottom. When trying to combine fractions w/ common denominators, you can use the denominator of both functions the common denominator because you are essentially multiplying by 1, which doesn't change the function

$$\frac{3}{x-1} - \frac{4}{3} = \frac{3}{x-1}\left(\frac{3}{3}\right) - \frac{4}{3}\left(\frac{x-1}{x-1}\right) = \frac{9-4(x-1)}{3(x-1)}$$

Difference Quotient

$\frac{f(x+h)-f(x)}{h}$; f(x)= $-3x^2+5x-1$

$$\frac{-3(x+h)^2+5(x+h)-1+3x^2-5x+1}{h} = \frac{-3(x^2+2xh+h^2)+5x+5h+3x^2-5x}{h}$$

$$= \frac{-3x^2-6xh-3h^2+5h+3x^2}{h} = \frac{-6xh-3h^2+5h}{h} = -3h-6x+5$$

✻ diff. quotient is quotient value of the difference btw the diff. values of x + h; just plug in (x+h) for x in the 1st part of the numerator, subtract by the original function, & divide the whole thing by h

Solving Inequalities

$$2x^2-3x-2 \le 0$$
$$(2x^2-4x)+(x-2) \le 0$$
$$2x(x-2)+(x-2) \le 0$$
$$(2x+1)(x-2) \le 0$$
$$(2x+1) \le 0$$
$$2x \le -1;\ x \le \frac{-1}{2}$$
$$(x-2) \le 0$$
$$x \le 2$$

Check the Chunks

$2(0)^2-3(0)-2 \le 0$ ✔

$2(-4)^2-3(-4)-2 \le 0$ ❌

$2(4)^2-3(4)-2 \le 0$ ❌

Solve Inequalities

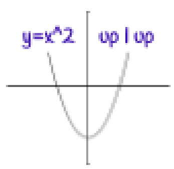
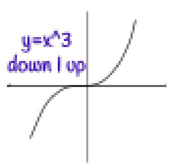
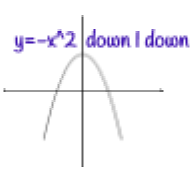
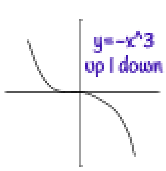
✻ factor out the function to make it easier to see the zeroes of the function; can use any factoring method

✻ once it's factored, then create equations (# of equations depends on # of roots) that set each root function in line w/ the inequality equal to 0

✻ then solve for x, making sure the flip the sign if multiplying or dividing by a negative number

✻ once they are solved, then plot the # on a number line and "check the chunks" which is to check where the domain falls for the inequality; as in where does the inequality hold true

End Behavior

	even	odd	
Up (+)			✻ if root function has even exponent (multiplicity), then graph bounces back in same
Down (–)			✻ odd multiplicity crosses axis ✻ leading degree (largest exponent) is # of roots

Holes

$$\frac{(x-4)(x+1)^2}{(x-4)(x+3)(x+5)^2}$$

There is a hole at x=4 which means that the function is not continuous at that point. The hole exists because the root function existed both in the numerator and denominator, thus could be canceled out, resulting in the hole