-- Rational Functions --

Operations

Adding/ Subtracting

$$\frac{2}{x-2} - \frac{5+2x}{x^2-4}$$
$$\frac{2}{x-2} - \frac{5+2x}{(x+2)(x-2)}$$
$$\frac{2}{x-2}\left(\frac{x+2}{x+2}\right) - \frac{5+2x}{(x+2)(x-2)}$$
$$\frac{2x+4}{x^2-4} - \frac{5+2x}{x^2-4} = \frac{-1}{x^2-4}$$

1. get common denominators by determining the lcm of the two denominators, and then multiplying each fraction by a fancy form of one
2. add/subtract the numerators and leave the denominators as is in order to get your final answer

Multiplying/ Dividing

$$\frac{4}{x-2} \div \frac{6+2x}{x^2-4}$$
$$\frac{4}{x-2} \cdot \frac{x^2-4}{6+2x}$$
$$\frac{2(2)}{x-2} \cdot \frac{(x+2)(x-2)}{2(3+x)}$$
$$\frac{2}{1} \cdot \frac{x+2}{3+x} = \frac{2x+4}{3+x}$$

1. if you are being asked to divide two fractions, KeepFlipChange
2. while multiplying, factor the parts of the fractions and cancel out
3. Multiply numerator • numerator and denominator • denominator to get your final answer

Solving Equations

$$\frac{6}{x+3} = \frac{4}{x-3}$$
$$(x-3)(x+3) \cdot \frac{6}{x+3} = \left(\frac{4}{x-3}\right) \cdot (x-3)(x+3)$$
$$6(x-3) = 4(x+3)$$
$$6x - 18 = 4x + 12$$
$$2x = 30$$
$$x = 15$$
$$\frac{1}{3} = \frac{1}{3} \checkmark$$

1. find the lcm of the denominators. then multiply both sides by that expression
2. simplify both sides of the equation
3. solve for x
4. check for extraneous solutions

$$\frac{x^2-20}{x^2-7x+12} = \frac{3}{x-3} + \frac{5}{x-4}$$
$$(x-3)(x-4) \cdot \frac{x^2-20}{(x-3)(x-4)} = \left(\frac{3}{x-3} + \frac{5}{x-4}\right) \cdot (x-3)(x-4)$$
$$x^2 - 20 = 3(x-4) + 5(x-3)$$
$$x^2 - 20 = 3x - 12 + 5x - 15$$
$$0 = x^2 - 8x + 7 = (x-7)(x-1)$$
$$x = 7 \quad \text{and} \quad x = 1$$
$$x = 7:\ \frac{29}{12} = \frac{29}{12} \checkmark \qquad x = 1:\ \frac{-19}{6} = \frac{-19}{6} \checkmark$$

Graphing

Horizontal Asymptote Rules

Big Bottom: ex. $\frac{2x+5}{x^2-4}$, HA is $y = 0$

Big Top: ex. $\frac{6x^3}{x+3}$, HA is y = quotient of fraction

Both Same: ex. $\frac{x^2-20}{x^2-7x+12}$, HA is $y = \frac{\text{leading coefficient}}{\text{leading coefficient}}$

Vertical Asymptote Rules

the denominator of a fraction is not allowed to equal 0. so if you **set the denominator equal to 0 and solve for x**, the vertical asymptotes are located at all the solutions

$y = \frac{9x}{x^2-16}$

Holes: none

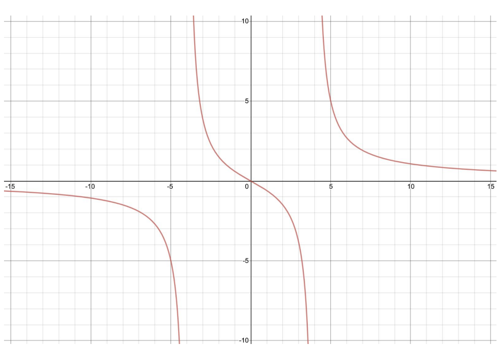
HA: big bottom, so $y = 0$

VA: $x^2 - 16 = 0 \rightarrow x = 4$ and $x = -4$

^^ note that they have odd multiplicities, so the graph will be "opposites" ^^

x-intercept: $0 = \frac{9x}{x^2-16} \rightarrow x = 0 \rightarrow (0,0)$

y-intercept: $y = \frac{9(0)}{0^2-16} \rightarrow y = 0 \rightarrow (0,0)$



^^ see how the graphs are going different ways on either side of the vertical asymptote. that means that that asymptote has an odd multiplicity. if they were even, they would be going the same direction ^^

1. simplify the fraction if necessary. note that the stuff you cancel out is the hole and that you need to plug it back into the new equation to solve for the y-value
2. find the horizontal asymptote, the vertical asymptotes, the x-intercepts, and the y-intercepts.
3. plot the points and lines on your graph
4. draw the graph based on what you already plotted in step 3. it may help you to learn about multiplicities if you usually have trouble eyeballing it. you can also plug in random points to make sure that you are graphing in the generally right direction

$y = \frac{2x^2+9x+10}{x^2-3x-10}$

$$y = \frac{(2x+5)(x+2)}{(x+2)(x-5)} \rightarrow y = \frac{2x+5}{x-5}$$

Holes: $(-2, -\frac{1}{7})$

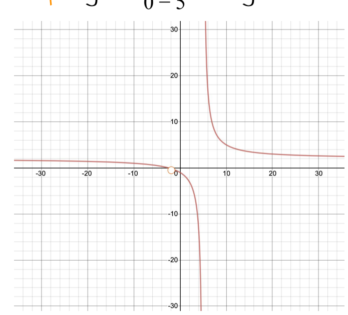
HA: both same, so $y = 2$

VA: $x - 5 = 0 \rightarrow x = 5$

^^ note that they have odd multiplicities, so the graph will be "opposites" ^

x-intercept: $0 = \frac{2x+5}{x-5} \rightarrow x = \frac{-5}{2} \rightarrow (\frac{-5}{2}, 0)$

y-intercept: $y = \frac{2(0)+5}{0-5} \rightarrow y = -1 \rightarrow (0,-1)$



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