-- Differential Equations --

Slope Fields

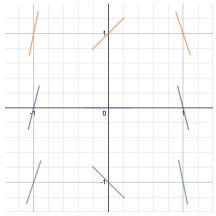
1. find the derivative of the function aka solve for y' of the differential equation
2. Plug in the points where you need to draw a line for the slope field
3. Draw the necessary line with the right slope at the corresponding point

$y' = y - 4x$

(0,0): $y' = 0$

(1,0): $y' = -4$ (0,1): $y' = 1$ (-1,0): $y' = 4$ (0,-1): $y' = -1$

(1,1): $y' = -3$ (-1,1): $y' = 5$ (-1,-1): $y' = 3$ (1,-1): $y' = -5$



Verifying Solutions

$y'' + y = 0$

is $y = C_1\sin x - C_2\cos x$ a solution?

$y' = C_1\cos x + C_2\sin x$

$y'' = -C_1\sin x + C_2\cos x$

$-C_1\sin x + C_2\cos x + C_1\sin x - C_2\cos = 0$ ✓

1. solve for y' and y'' if necessary. remember to include a $\frac{dy}{dx}$ if you take the derivative of y

2. plug it back into the differential equation to see if it satisfies the rule

$-xy' + y^2y'' = -1$

is $3x^2 + 3y^2 = C$ a solution?

$6x + 6y\frac{dy}{dx} = 0 \rightarrow y' = -x/y$

$y'' = \frac{x}{y^2} \cdot \frac{dy}{dx} - \frac{1}{y} \rightarrow y'' = \frac{-x^2}{y^3} - \frac{1}{y} = \frac{-x^2-y^2}{y^3}$

$-x(\frac{-x}{y}) + y^2(\frac{-x^2-y^2}{y^3}) = -1$ ✓

Separation of Variables

$yy' - 2e^x = 0$ / $y(0) = 3$

$ydy = 2e^xdx$

$\int ydy = \int 2e^xdx$

$\frac{1}{2}y^2 = 2e^x + C$

$\frac{1}{2}(3)^2 = 2e^0 + C$

$\frac{9}{2} = 2 + C$

$C = \frac{5}{2}$

$y = \sqrt{4e^x + 5}$

1. Replace any y' with $\frac{dy}{dx}$. Put all the dy and y on one side of the equation and the dx and x on the other side.

2. integrate both sides

3. manipulate/simplify if necessary

4. Plug in the initial condition if you are given one. If not, skip to step 6

5. Solve for the constant and plug it back into the equation

6. Solve for y

$y'\cos^2x + y - 1 = 0$ / $y(0) = 5$

$\frac{1}{dx}\cos^2x = (1-y)\frac{1}{dy}$

$\int \frac{1}{\cos^2x}dx = \int \frac{1}{1-y}dy$

$\int \sec^2xdx = -\int \frac{1}{u}du$

$\tan x + C = -\ln|1-y|$

make all terms the exponent to the base e

$e^{\tan x + C} = \left|\frac{1}{1-y}\right|$

$\pm e^Ce^{\tan x} = \frac{1}{1-y}$

$C_1 = \pm e^C$

$C_1e^{\tan x} = \frac{1}{1-y}$

$C_1e^{\tan 0} = \frac{1}{1-5}$

$C_1 = -\frac{1}{4}$

$-\frac{e^{\tan x}}{4} = \frac{1}{1-y}$

$-4 = e^{\tan x} - ye^{\tan x}$

$y = \frac{4 + e^{\tan x}}{e^{\tan x}}$

$y = \frac{4}{e^{\tan x}} + 1$