-- Polynomial Functions --

Classifying

Based on Highest Degree

$x^0 \rightarrow$ constant; $x \rightarrow$ linear; $x^2 \rightarrow$ quadratic; $x^3 \rightarrow$ cubic; $x^4 \rightarrow$ quartic

Based on Number of Terms

1 term → monomial; 2 terms → binomial; 3 terms → trinomial

* to review adding, subtracting, multiplying, and dividing polynomials, reference the chapter 1 study guide

Identities

Squaring Binomials

$(a+b)^2 = a^2+2ab+b^2$ $(a-b)^2=a^2-2ab+b^2$

$(3x^3 - 5x)^2 = 9x^6 - 30x^4 + 25x^2$

$x^2 + 6x + 9 = (x + 3)^2$

Difference of Squares

$(a + b)(a - b) = a^2 - b^2$

$4x^2 - 9 = (2x + 3)(2x - 3)$

$(2x^3 + 5)(2x^3 - 5) = 4x^6 - 25$

Difference/Sum of Cubes

$(a + b)^3 = (a + b)(a^2 - ab + b^2)$

$(a - b)^3 = (a - b)(a^2 + ab + b^2)$

Same ↑ Opposite ↑ Always ↑

Positive

Successive Differences

x	y
0	3
1	5
2	13
3	33
4	71
5	133

1	2	3
> 2	> 6	> 6
> 8	> 12	> 6
> 20	> 18	> 6
> 38	> 24	
> 62		

⟹ cubic

The function is a cubic with a degree of 3

Sketching Graphs

End Behavior

if the highest degree is even:
end behavior points in the same direction
+ coefficient ↑ ↑ − coefficient
↓ ↓

if the highest degree is odd:
end behavior points in different directions
+ coefficient ↓ ↑ − coefficient
↑ ↓

example

$f(x) = -x^3 + x^2 + 6x - 6$

The end behavior is ↑ ↓

Intercepts

for x-intercepts, set the function equal to 0 and factor.
for y-intercepts, let x = 0 and solve

example

$f(x) = -x^3 + x^2 + 6x - 6$

$0 = (-x^3 + x^2) + (6x - 6)$

$0 = -x^2(x - 1) + 6(x - 1)$

$0 = (-x^2 + 6)(x - 1)$

$0 = -x^2 + 6$ $0 = x - 1$

$6 = x^2$ $1 = x$

$x = \pm\sqrt{6}$

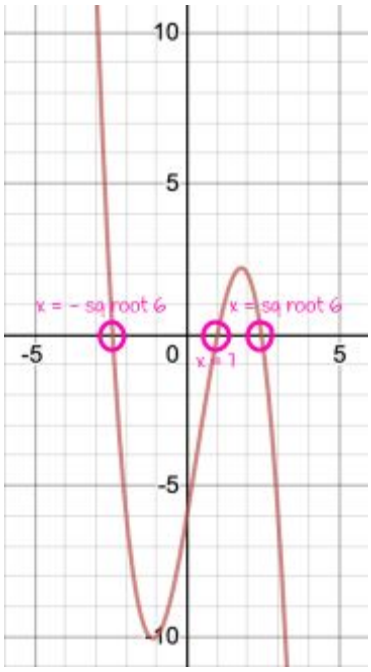
so this function intercepts the x-axis at $x = \sqrt{6}, -\sqrt{6}$, and 1

all the zeroes have an odd multiplicity (since they are to the power of one in the factored form), so the line goes through them. if they had an even multiplicity, the function would "bounce off" the x-axis.

$f(0) = -6$

so this function intercepts the y-axis at $y = -6$

Sketching



don't worry about how high the "bumps" are. just focus on getting the end-behavior and intercepts right