-- Sequences and Series --

Basics

Arithmetic Sequences

- use when there is a constant difference between each term, aka you can add/subtract a number to get to the next number in the sequence
- ex. 3, 6, 9, 12, 15, 18
- explicit: $a_n = a_1 + (n - 1)d$
- recursive: $a_n = a_{n-1} + d$

Geometric Sequences

- use when there is a constant ratio between each term, aka you can multiply/divide a number to get to the next number in the sequence
- ex. 1, 3, 9, 27, 81, 243
- explicit: $a_n = a_1 \cdot r^{n-1}$
- recursive: $a_n = r \cdot a_{n-1}$

Geometric Series

- the sum of a geometric sequence
- $\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$
- ex. $\sum_{k=1}^{4} k^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$

Practice Problems

an arithmetic sequence has the terms $a_{21} = 10.2$ and $a_{25} = 11.0$. the 1st term is a_1. find explicit and recursive formula

$a_{21} + (25 - 21)d = a_{25}$

$4d = 11 - 10.2 \rightarrow d = \frac{.8}{4} = .2$

so recursive formula is $a_n = a_{n-1} + .2$

$10.2 = a_1 + .2(21 - 1) \rightarrow a_1 = 6.2$

or

$11.0 = a_1 + .2(25 - 1) \rightarrow a_1 = 6.2$

so explicit formula is $a_n = 6.2 + .2(n - 1)$

1. find the d/r by setting up an equation as shown
2. make your recursive formula.
3. set up an equation using the explicit formula to solve for a_0/a_1. note that when the first term is a_0, "n−1" does not appear, it is only n
4. set up the explicit formula

a geom. sequence has terms $a_4 = -1.56$ and $a_7 = .01248$. the 1st term is a_0 find the explicit and recursive formula

$a_4 \cdot r^{7-4} = a_7$

$r^3 = \frac{.01248}{-1.56} \rightarrow r = \sqrt[3]{-.008} = -0.2$

so recursive formula is $a_n = -0.2 \cdot a_{n-1}$

$-1.56 = a_0 \cdot (-0.2)^4 \rightarrow a_0 = -975$

or

$.01248 = a_0 \cdot (-0.2)^7 \rightarrow a_0 = -975$

so explicit formula is $a_n = -975 \cdot -0.2n$

write the following geom. series in summation notation

$\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} + \frac{1}{162} + \frac{1}{486}$

n = # terms = 6 terms

a = first term = $\frac{1}{2}$

$r = \frac{1}{6} \div \frac{1}{2} = \frac{1}{3}$

$\sum^{n-1} a \cdot r^n \rightarrow \sum_{k=n}^{5} \frac{1}{2}\left(\frac{1}{3}\right)^k$

Loan Payments

Definition

for problems with money, use $F = P(1 + \frac{r}{n})^{nt}$ where F is future value, P is the initial value, r is interest rate, t is time in years, and n is how many times its compounded in one year (ex. quarterly is 4, monthly is 12)

Practice Problems

Jeff deposited \$15,00 into an account w/ a 5.1% interest rate that is compounded daily. How much is it worth after 72 months?

** just forget leap years exist for this :) **

$F = 15{,}000(1 + \frac{.051}{365})^{(365)(6)}$

$F = \$20{,}369.30$

1. read the problem and try to determine what number corresponds with what variable. then set up your equation
2. solve the equation
3. if the problem asks for ___ payments, divide your value by how many times they have to pay in the given time span

Bartholomew bought a boat for \$19,000 with a 3.5% interest rate compounded semi-annually for 5 years. What is his monthly payment?

$F = 19{,}000(1 + \frac{.035}{2})^{(2)(5)}$

$F = \$22{,}599.46$

$MP = \frac{22{,}599.46}{60} = \$376.66$