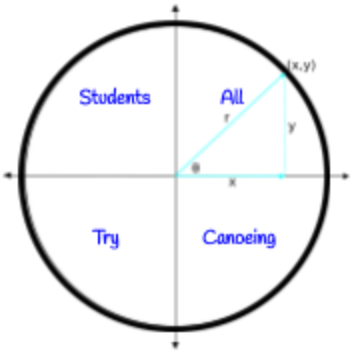
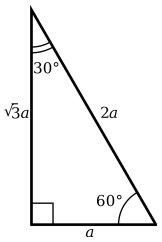
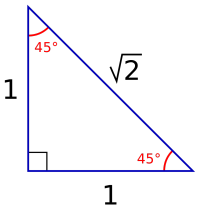
-- Basic Triangle Trigonometry --

Trig



All: All trig functions are positive
Students: Sine/cosecant is positive
Try: Tangent/cotangent is positive
Canoeing: Cosine/secant is positive





SOH CAH TOA

$\sin = \frac{opposite}{hypotenuse} = \frac{y}{r}$ $\csc = \frac{1}{\frac{opposite}{hypotenuse}} = \frac{r}{y}$

$\cos = \frac{adjacent}{hypotenuse} = \frac{x}{r}$ $\sec = \frac{hypotenuse}{adjacent} = \frac{r}{x}$

$\tan = \frac{opposite}{adjacent} = \frac{y}{x}$ $\cot = \frac{adjacent}{opposite} = \frac{x}{y}$

**Ex: $\sin\theta = \frac{\sqrt{7}}{2}$, $\cos\theta = \frac{3\sqrt{7}}{2}$

Find value of other trig functions

$$\csc\theta = \frac{1}{\sin\theta} = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{2}{3\sqrt{7}} = \frac{2\sqrt{7}}{21}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{7}}{2}}{\frac{3\sqrt{7}}{2}} = \frac{2\sqrt{7}}{6\sqrt{7}} = \frac{2}{6} = \frac{1}{3}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{3\sqrt{7}}{2}}{\frac{\sqrt{7}}{2}} = \frac{6\sqrt{7}}{2\sqrt{7}} = \frac{6}{2} = 3$$

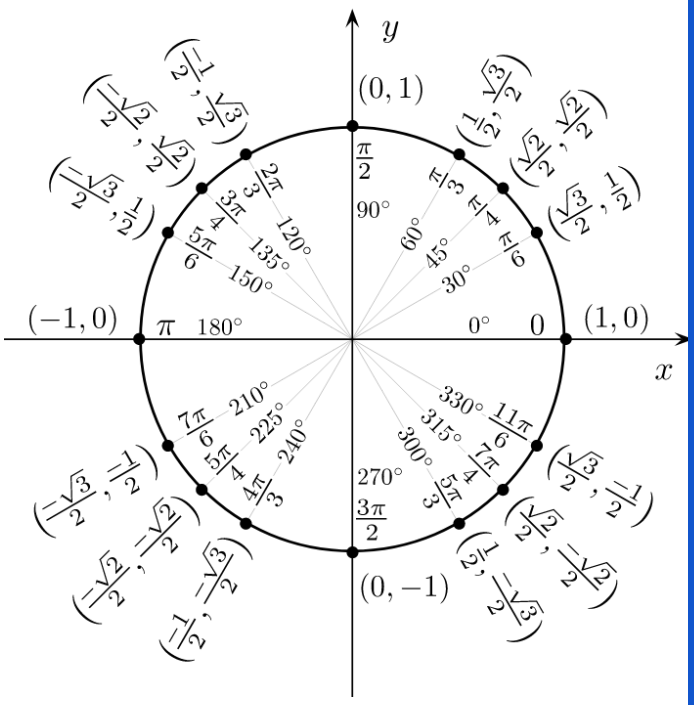
Unit Circle

sin = y csc = 1/y
cos = x sec = 1/x

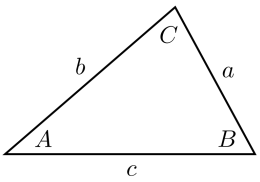
Trig Identities

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$\cot^2\theta + 1 = \csc^2\theta$

Unit Circle



Solving Non-Right Triangles:



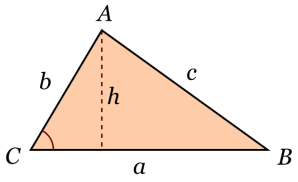
Law of Sines

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

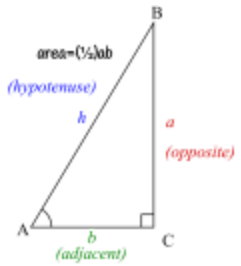
Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cdot cosA$
$b^2 = a^2 + c^2 - 2ac \cdot cosB$
$c^2 = a^2 + b^2 - 2ac \cdot cosC$

Area of a Triangle



$$\frac{ah}{2} = \frac{ab\sin C}{2}$$



Even-Odd Identities

Odd	Even	Odd
$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$

$$\sin(\arccos x) = \sqrt{1-x^2} \qquad \tan(\arcsin x) = \frac{x}{\sqrt{1-x^2}}$$

$$\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}} \qquad \tan(\arccos x) = \frac{\sqrt{1-x^2}}{x}$$

$$\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}} \qquad \cot(\arcsin x) = \frac{\sqrt{1-x^2}}{x}$$

$$\cos(\arcsin x) = \sqrt{1-x^2} \qquad \cot(\arccos x) = \frac{x}{\sqrt{1-x^2}}$$

Formulas

✱Converting Decimals, Degrees, Minutes, & Seconds

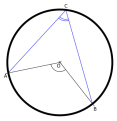
$\sim 1^o = 60'$ (60 minutes)
$\sim 1' = 60''$ (60 seconds)

$$\sim \frac{60mi}{1hr} \cdot \frac{5280ft}{1mi} \cdot \frac{12in}{1ft} \cdot \frac{1hr}{60min} \cdot \frac{1min}{60sec} = 1056\,\frac{in}{sec}$$

**Convert $32^0 4'22''$ to a decimal in degrees

$$32^0 + 4' \cdot \frac{1^0}{60} + 22'' \cdot \frac{1'}{60^4} \cdot \frac{1^0}{60'} = 32.07^0$$

✱Arc Length: $l = \theta r$
✱Area of a Sector: $A = \frac{1}{2}\theta r^2$
✱Arc Measure: $m < ACB = m\widehat{AB}$



✱Degrees to Radians:
$\theta \cdot \frac{\pi}{180} = radians$
✱Radians to Degrees:
$r \cdot \frac{180}{\pi} = \theta(degrees)$
✱Linear Speed: $v = \frac{l}{t} = \frac{\theta r}{t} = wr$
(length unit per unit time)
✱Angular Speed: $w = \frac{\theta}{t}$ (rad/time)

** Examples:

a. $cos135^o = -\frac{\sqrt{2}}{2}$
b. $sin\frac{-2\pi}{3} = -\frac{\sqrt{3}}{2}$
c. $tan(-675^o) = tan(45^o) = 1$
d. $sec\frac{7\pi}{6} = -\frac{2\sqrt{3}}{3}$
e. $csc\frac{11\pi}{4} = \sqrt{2}$
f. $cot\frac{-5\pi}{6} = \sqrt{3}$
g. $sin(cos^{-1}\frac{1}{2}) = sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
h. $tan^{-1}(cos\pi) = -\frac{\pi}{4}$
i. $cos(sin^{-1}\frac{1}{2}) = \frac{\sqrt{3}}{2}$
j. $sin^{-1}(sin\frac{7\pi}{4}) = -\frac{\pi}{4}$
k. $sin(tan^{-1}\sqrt{3}) = \frac{\sqrt{3}}{2}$
l. $arccos(1) = 2\pi$